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SHOCK WAVE STRUCTURE IN A MIXTURE OF GAS AND MELTING PARTICLES

A. V. Fedorov and V. M. Fomin

UDC 532.529.5

The process of fusion of solid particles dispersed within a gas flow occurs during various gasdynamic reactions in technical equipment. In particular, for noncalculable regimes in Laval nozzles, using a gas with solid combustion products of some fuel as a working medium, the structure of the shock wave which develops is complicated as compared to that of a shock wave propagating in a mixture of gas and particles without consideration of the phase transition, because aside from the relaxation process of equalization of the temperatures of the phases, particle transition from the solid to the liquid state occurs with a finite relaxation time. The driving force for this transition is the difference of the liquid concentration from its equilibrium value.

The structure of a shock wave in a mixture of gas with melting particles was studied in [1, 2] within the framework of single-velocity, single-temperature mechanics of heterogeneous media with consideration of the nonequilibrium fusion process. The case in which the process of heat exchange between the phases occurs at a finite rate requires consideration based on a model which considers the difference between the temperatures of the phases. At the same time, considering the particles to be sufficiently small, and assuming that they are instantaneously carried off by the gas flow, we may conclude that the simplified model proposed in [3] will be adequate for the study of shock wave propagation in a mixture of gas with metal particles, with consideration of fusion. We will assume that the heat of phase transition L is independent of the fusion temperature, determined by the pressure of the mixture.

1. Formulation of the Problem of Determination of Shock Wave Structure in the Mixture. Study of the Hugoniot Adiabat. We will consider the process of shock wave propagation in a mixture of gas and solid particles. The equations describing this phenomenon in a reference frame traveling with the shock wave have the form

$$\rho u = c_1,\ p + c_1 u = c_2,\ e + pv + u^2/2 = c_3, \quad (1.1)$$
$$p = \frac{\overline{\alpha} RT}{w},\ e = \bar{c}_{V1} T + c_* T_2 + L\xi,$$
$$u\dot{\xi} = \varkappa,\ u\dot{T}_2 = q,\ \rho_{22} = \rho_{33} = r$$

(using the notation of [3]). We will use the source function

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$$\varkappa = -\frac{1}{\tau}(\xi - \xi_e(S, v)), q = -\frac{1}{\tau_1}(T_2 - T), \xi_e = \xi_0 e^{-\frac{L}{\bar{c}_{V1}}(T^{-1} - T_0^{-1})}.$$

System (1.1) is complemented by the boundary conditions

$$u = u_0, v = v_0, T = T_2 = T_0, \xi = \xi_0, \dot{u}, \ldots, \dot{\xi} \to 0, \quad (1.2)$$
$$x \to -\infty, u = u_f, v = v_f, T = T_2 = T_f, \xi = \xi_f,$$
$$\dot{u}, \ldots, \dot{\xi} \to 0, x \to +\infty,$$

where values with the subscript f are the flow parameters of the mixture in its final state. This state is represented by a point on a completely equilibrium Hugoniot adiabat

$$c_1^2 v^2/2 + \bar{c}_{V1} T + pv + L\xi_0 \exp(-L(T^{-1} - T_0^{-1})/\bar{c}_{V1}) = \quad (1.3)$$
$$= \bar{c}_{V1} T_0 + p_0 v_0 + L\xi_0 + c_1^2 v_0^2/2, \; T = pw/\bar{\alpha}R, \; w = v - \beta.$$

Types of mixture flow in the presence of fusion of a discrete phase $\Gamma(\tau, \tau_1)$ were presented in [3].

Definition 1. The Hugoniot adiabat for the flow $\Gamma(0, 0)$ is the completely equilibrium adiabat, while for the flows $\Gamma(0, \infty)$, $\Gamma(\infty, 0)$ it is partially equilibrium.

We write the equilibrium adiabat (1.3) defining the final state in the form

$$A(v) = \bar{c}_V T + pv + c_1^2 v^2/2 - \bar{c}_V T_0 - p_0 v_0 - c_1^2 v_0^2/2 = L\xi_0 (1 - \exp(-L(T^{-1} - T_0^{-1})/\bar{c}_{V1})) = B(v). \quad (1.4)$$

If $B(v) = 0$, then Eq. (1.4) gives the final state in the flow $\Gamma(\infty, 0)$. We define this expression by solution of an equation quadratic in v [to which Eq. (1.4) transforms in the given case]:

$$\frac{1 + \gamma_T}{2(1 - \gamma_T)} c_1^2 v^2 + \left(\frac{\gamma_T}{\gamma_T - 1} c_2 - \beta c_1^2 \frac{1}{\gamma_T - 1}\right) v - \left[\frac{p_0 v_0 \gamma_T}{\gamma_T - 1} + u_0^2 \left(\frac{1}{2} + \frac{m_{20} + m_{30}}{\gamma_T - 1}\right)\right] = 0. \quad (1.5)$$

One of its solutions is v_0, the other being

$$v = v_f^{\infty,0} = v_0 \left[\frac{\gamma_T - 1}{\gamma_T + 1} + \frac{2}{\gamma_T + 1}\left(\frac{m_{10}}{M_{T,0}^2} + m_{20} + m_{30}\right)\right], \quad (1.6)$$

$$M_{T,0}^2 = u_0^2 \left(\frac{\gamma_T p_0 v_0^2}{w_0}\right)^{-1}, \; \gamma_T = \bar{c}_p/\bar{c}_V, \; \bar{c}_V = \bar{c}_{V1} + \bar{c}_*, \; \bar{c}_p = \bar{c}_{p1} + \bar{c}_*, \; \bar{c}_{V1} = \bar{\alpha} c_{V1},$$
$$\alpha = (\rho_{20} + \rho_{30})/\rho_0, \; \bar{c}_* = \alpha c_*,$$

whence it is evident that as $m_{20} + m_{30} \to 0$, Eq. (1.6) coincides with the gasdynamics result, with $v_f^{\infty,0} < v_0$. Thus, A(v) has the form of an inverted parabola.

We will consider the function $T = T(v) = (c_2 - c_1^2 v)w/\bar{R}$ ($\bar{R} = \bar{\alpha}R$), which has a maximum at the point $v = v_* = v_0(1 + m_{20} + m_{30} + m_{10}/\gamma_f M_{f0})/2$. It can be shown that $v_* < v_0$ given the condition $1 < \gamma_f M_f^2$. Since $v_* - v_f^{\infty,0} = \frac{(3 - \gamma_T) m_{10}}{2(1 + \gamma_T)}\left(1 + \frac{1 - 3\gamma_T}{\gamma_T (3 - \gamma_T) M_{T,0}^2}\right) > 0$ for $M_{T,0}^2 >$ $(3\gamma_T - 1)/\gamma_T(3 - \gamma_T) \equiv M_*^2$, then at $M_{T,0}^2 < M_*^2 v_* < v_f^{\infty,0} < v_0$ and in the interval $(v_f^{\infty,0}$ $v_0) dT/dv < 0$.

The quantity $dB/dv = \exp(-L(T^{-1} - T_0^{-1})/\bar{c}_{V1}) L dT/dv/\bar{c}_{V1} T^2$ at the point $v = v_0$ is positive, since $dT/dv > 0$, the function B(v) is continuous over the interval (v_*, v_0); therefore, on the basis of the above, A(v) and B(v) intersect only at a single point to the left of $v_f^{\infty,0}$ i.e., $v_f^{0,0} < v_f^{\infty,0} < v_0$. We will formulate this result as:

Statement 1. For flow of an equilibrium mixture of gas and solid particles with consideration of fusion of type $\Gamma(0, 0)$ the final state u_f, v_f, ..., ξ_f on the adiabat Eq. (1.3) is uniquely determined.

To proceed further we must determine the relative magnitudes of $v_f^{0,0}$, v_*. Evaluation of $v_f^{0,0}$ in the general case is difficult, so we will perform an approximate evaluation. Inasmuch as $v_0 - v_f \ll 1$ from a priori assumptions, the following operations are highly accurate.

We take the Taylor series $B(v) = B_0(T - T_0) + O(T - T_0)^2$, $B_0 = \bar{L}^2\xi_0\bar{c}_{V1}$, $\bar{L} = L/\bar{c}_{V1}T_0$; substituting in Eq. (1.4), we obtain an analog of Eq. (1.5) for $\Gamma(0, 0)$, the solution of which has the form of (1.6), where instead of γ_T we use $\gamma_0 = c_p^0/c_V^0$, $c_p^0 = \bar{c}_p + B_0$, $c_V^0 = \bar{c}_V + B_0$, $M_{0,0}^2 = (\gamma_0 p_0 v^2_0/w_0)^{-1}u_0^2$. We may demonstrate in a similar manner that $v_* < v_f^{0,0} < v_f^{\infty,0} < v_0$ for $M_{0,0}^2 < M_*$, M_* being defined by γ_0.

2. Formulation of the Problem in the Plane (u, ξ). Study of Singular Points. Formulation of the Main Result. Choosing as the unknown functions in Eq. (1.1) (u, ξ), and using the final relationships of Eq. (1.1), we obtain

$$\frac{du}{dx} = -\frac{a\varkappa + bq}{uc} = \mathscr{P}(u, \xi), \quad \frac{d\xi}{dx} = \frac{\varkappa}{u} = Q(u, \xi), \tag{2.1}$$

where $a = (e_{vs}e_\xi - e_{v\xi}e_s)/e_s$; $b = (e_{vs}e_{T_2} - e_{vT_2}e_s)/e_s$; $c = (u^2 - c_f^2)/c_1 v^2$. The singular points of system (2.1) are its equilibrium positions, and as was shown in Sec. 1, with M_0 limited, two such points exist (u_0, ξ_0), (u_f, ξ_f). Then the problem formulated above of determining a solution to Eqs. (1.1), (1.2) reduces to finding a solution of Eq. (2.1) satisfying the steady-state conditions at $\pm\infty$.

The possibility of solving this problem depends on the type of singular points involved, which we find by analyzing the roots of the corresponding characteristic equation

$$\lambda^2 + \lambda\sigma + \Delta = 0, \tag{2.2}$$

$$\sigma = \mathscr{P}_u + Q_\xi = -(u^2 - c_f^2 + v^2 p_\xi \xi_{e,v})/\tau c_1 v^2 (u^2 - c_f^2) -$$

$$-(u^2 - c_f^2 + v^2 p_{T_2} T_{2e,v})/\tau_1 c_1 v^2 (u^2 - c_f^2), \ \Delta = \mathscr{P}_u Q_\xi - Q_u \mathscr{P}_\xi = (u^2 - c_f^2)(u^2 - c_e^2)/u^2\tau\tau_1$$

(σ, Δ are taken at the initial and final points). We will consider the roots of Eq. (2.2) for the condition $c_e < u < c_f$. Then $\Delta < 0$, whence we find the different real roots

$$\lambda_{1,2} = (\sigma \pm \sqrt{\sigma^2 - 4\Delta})/2.$$

Hence it is evident that for the condition $u \in (c_e, c_f)$ the singular point is a saddle.

Let $u > c_f$, $u < c_e$. In analogy to [4], we will consider the radicand $D_2 = \sigma - 4\Delta$ as a second-degree polynomial in $\zeta = \tau_1/\tau$:

$$D_2(\zeta) = \tau^2 \{(u^2 - c_\xi^2)^2\zeta^2 + 2\zeta[(u^2 - c_\xi^2)(u^2 - c_{T_2}^2) - 2(u^2 - c_f^2)(u^2 - c_e^2)] + (u^2 - c_{T_2}^2)^2\},$$

$$c_\xi^2 = c_f^2 - v^2 p_\xi \xi_{e,v}, \ c_{T_2}^2 = c_f^2 - v^2 p_{T_2} T_{2e,v}.$$

Its roots are defined by

$$\zeta_{1,2} = \frac{-(u_\xi u_T - 2u_f u_e) \pm \sqrt{(u_\xi u_T - 2u_f u_e)^2 - u_\xi^2 u_T^2}}{u_\xi^2}, \tag{2.3}$$

where $u_\xi = u^2 - c_\xi^2$; $u_T = u^2 - c_{T_2}^2$; $u_f = u^2 - c_f^2$, $u_e = u^2 - c_e^2$. The discriminant $D_1 = 4u_f u_e(u_f u_e - u_\xi u_T) = 4(u^2 - c_f^2)(u^2 - c_e^2)p_\xi \xi_{e,v} p_{T_2} T_{2e,v}$.

We will establish the sign of the quantities p_ξ, $\xi_{e,v}$, p_{T_2}, $T_{2e,v}$:

$$\frac{\partial T_{2e}}{\partial v} = \frac{p\dot{\varphi}_2\dot{\varphi}_1}{\bar{c}_{V1}T\Delta_1}, \ \Delta_1 = \dot{\varphi}_2^2\ddot{\varphi}_1 + \dot{\varphi}_1^2\ddot{\varphi}_2 + \ddot{\varphi}_1\ddot{\varphi}_2 = D(D + \xi \ln^2(\xi/\xi_p) + 1) > 0,$$

$$\dot{\varphi}_1 = \ln(\xi/\xi_p), \ \xi_p = \xi_0 \exp(L/\bar{c}_{V1}T_0), \ \ddot{\varphi}_1 = \xi^{-1}, \ \dot{\varphi}_2 = -D/T_2, \ \ddot{\varphi}_2 = D/T_2^2, \ D = \bar{c}_*/\bar{c}_{V1}.$$

Wherefore $\frac{\partial T_{2,e}}{\partial v} < 0$. The quantities $\frac{\partial \xi_e}{\partial v} = \frac{p\dot{\varphi}_1\dot{\varphi}_2}{\bar{c}_{V1}T\Delta_1} = \frac{pD}{\bar{c}_{V1}T\Delta_1 T_2^2}\ln(\xi/\xi_p)$, $p_\xi = p\ln(\xi/\xi_p)$, $p_{T_2} = -p\frac{D}{T_2}$.

Hence $p_\xi \xi_{e,v} p_{T_2} T_{2e,v} = -T_{2e,v}\frac{p^3 D^2}{\bar{c}_{V1}T\Delta_1 T_2^3}\ln^2(\xi/\xi_p) > 0$ always, while $D_1 < 0$ and, consequently, $D_2 > 0$.

This means that the roots (9) of the equation for λ are different and real; for $\sigma < 0$, $\lambda_2 < \lambda_1 < 0$, for $\sigma > 0$, $\lambda_1 > \lambda_2 > 0$.

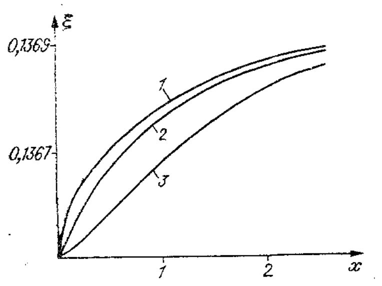

Fig. 1

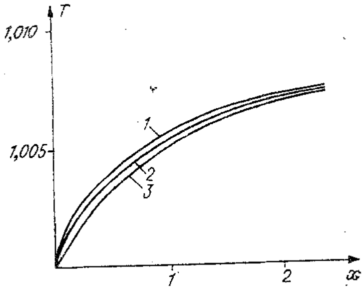

Fig. 2

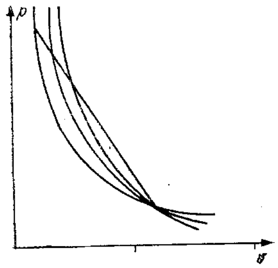

Fig. 3

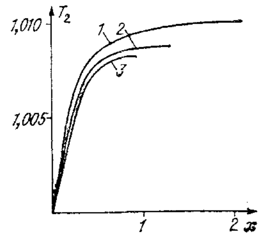

Fig. 4

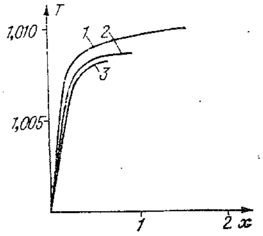

Fig. 5

The relative magnitude of the quantities $c_{T_2}{}^2$, $c_\xi{}^2$ are of interest, so we find $c_{T_2}{}^2 - c_\xi^2 = \frac{p^2}{\bar{c}_{V1} T \Delta_1} \frac{D^2}{T_2^2 \xi}\left(\frac{\xi}{D}\ln^2\frac{\xi}{\xi_p} - 1\right)$. Calculating this quantity at the equilibrium points, where $\xi_e = \xi_e(S, v)$, we find $\frac{\xi}{D}\ln^2\frac{\xi}{\xi_p} - 1 \equiv \xi_0 e^{-(\bar{L}-\bar{L}_0)} \frac{\bar{L}^2}{D} - 1$. It is evident that for appropriate parameter values we may have $c_{T_2}{}^2 \gtrless c_\xi{}^2$. The values of $m_{1,0}$, ξ_0 actually chosen are such that $c_e{}^2 < c_{T_2}{}^2 < c_\xi{}^2 < c_f{}^2$.

Thus, let $u_{0,f} > c_f$, then $\sigma > 0$, and for all $u_{0,f}$ the given singular point is a node with negative eigenvalues. The equality $\sigma = 0$ is achieved by corresponding choice of τ, τ_1. If $u_{0,f} < c_{e,0,f}$, then $\sigma < 0$, which means that the singular point is a node with negative eigenvalues $\lambda_{1,2} < 0$. The above analysis permits the following statement.

Statement 2. If the value u at which $\mathcal{P}$ and Q vanish lies in the interval (c_e, c_f), then the singular point of Eq. (7)† is a saddle, while if $u > c_f$ and $u < c_e$, the singular point is a node with negative eigenvalues.

Further analysis of the problem reduces to study of the integral curves passing through the singular points 0 and f, as was done in [5]. A unique situation may then arise, based on the presence of a singular line $u^2 - c_f{}^2(u) = 0$. It is simple to obtain an explicit solution of this equation in the form $u/u_0 = u_m/u_0 \equiv (m_{10} + M_{f,0}{}^2(\gamma_f + 1 - m_{10}))/(1 + \gamma_f)M_{f,0}{}^2$. Hence it is evident that the value of the velocity at the sonic point $u = u_M$, treated as a function of $M_{f,0}$, is greater than unity for $M_{f,0} < 1$ and less than unity in the opposite case.

We will now make use of the result of [7] concerning the behavior of the function u(x) in the definition region. It is simple to prove that the derivative with respect to T of the equilibrium value of the corresponding parameter $\xi = \xi_e$, $T_2 = T_{2e}$ is positive, so that (i) and (ii) from Sec. 2 of [7] are valid for our case also. Neglecting the volume concentration of particles $m_3 + m_2 \ll 1$ the equation of state takes on the form pv = RT. Then given condition (ii) of [7], $u_0 < c_{f,0}$ u(x) either decreases monotonically to the final state, or has an initial local minimum. This means that u = u(x), decreasing for $u_0 < c_{f,0}$, cannot reach the value $u = u_M$. However, the point u_f is attainable, since at $+\infty$, $\lambda_i < 0$ for i = 1, 2. Then $u_0 > c_{f,0}$(i), and for continuous change of u(x) we find the point $u = u_M$, where $du/dx \sim \infty$. Therefore, we have a discontinuity from u_0 to $\tilde{u}$ [7] with either a monotonic zone for u(x) or a local minimum. As a result we may formulate Statement 3.

Statement 3. The solution of the boundary problem for system (7)† exists in the class of continuously differentiable functions for the condition $u_0 \in (c_{e,0}, c_{f,0})$. If $u_0 > c_{f,0}$,

†As in Russian original — Publisher.

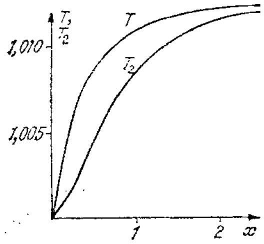

Fig. 6

there is a generalized solution of the boundary problem in the form of a discontinuity, complemented by a zone of continuous change for the functions u, v, ...

3. Example of Numerical Calculation. Evaluation of Results. The numerical algorithm developed by the authors for solution at $u_0 \in (c_{e,0}, c_{f,0})$ consists of the following steps. On the basis of Statement 3, in the given case the singular point is a saddle; therefore, to reach the equilibrium state at $-\infty$ it is necessary to choose a solution of the linearized system corresponding to $\lambda_1 > 0$. Taking

$$u - u_0 = h_{11}e^{\lambda_1 x},\ \xi - \xi_0 = h_{12}e^{\lambda_1 x},$$

in the solution, where (h_{11}, h_{12}) is the eigenvector corresponding to the eigenvalue λ_1, $u = u_0 - \varepsilon$, we find $x_\varepsilon = (1/\lambda_1)\ln(-\varepsilon/h_{11})$, and then $\xi_e = \xi_0 + h_{12}e^{\lambda_1 x_\varepsilon}$. We then solve the Cauchy problem for Eq. (2.1) with initial values $u = u_0 - \varepsilon$, $\xi = \xi_\varepsilon$, $x = x_\varepsilon$. Integration is then performed by the method of [6] to $u = u_f + \varepsilon_1$.

For $u_0 > c_{f,0}$ we determine the value $u = \tilde{u}$, the mixture velocity behind the discontinuity in the frozen shock wave, and then integrate the Cauchy problem $u = \tilde{u}$, $\xi = \xi_0$, $x = 0$. The computation is halted at $u = u_f + \varepsilon_1$. As dimensionless variables we use $u = u/\sqrt{RT_0}$, $p = p/p_0$, $\rho = \rho/\rho_{11,0}$, $p_0 = \rho_{11,0}RT_0$, x/x_0, $t = t/\tau$, $x_0 = \sqrt{RT_0}\tau$, $T = T/T_0$, $T_2 = T_2/T_0$.

In performing numerical calculations of the frozen shock wave structure the effect of relaxation time τ on the distribution of relative mass concentration ξ over the length of the wave was studied (Fig. 1, curves 1-3 correspond to $\tau = 0.01, 0.1, 0.5$). Ahead of the wave the mixture is in an equilibrium state at the fusion temperature. The particles traverse the wave with constant parameters $\xi = \xi_0$, $T_2 = T_{20} = 1$. The gas temperature within the shock wave increases such that heat exchange begins between the continuous phase and the dispersed phase, with resulting fusion of solid phase particles. This fusion occurs by a mechanism (the driving force of which is the difference of the concentration from its equilibrium value), characterized by a relaxation time τ. It is evident that with decrease in τ the process of transition of ξ to the equilibrium value ξ_f accelerates. With increase in τ at some τ_* saturation occurs, so that at $\tau > \tau_*$ the ξ process can be considered frozen. The behavior of gas temperature is similar (Fig. 2, τ_1 = const, curves 1-3 correspond to $\tau = 0.01, 0.1, 0.5$), T_2 changes less. This can apparently be explained by the fact that in the given case the dominant role is played by the ξ-process, which is determined directly by gas temperature. At the same time, heat exchange between the discrete and gaseous phases is prolonged, in view of τ_1 = const = 1, so that the temperature of the discrete phase reacts more weakly to change in τ.

The effect of the heat of phase transition on the flow pattern is of interest. Increase in L leads to an increase in the temperatures of the phases at corresponding points along the relaxation zone, since with increase in L the equilibrium speed of sound decreases, i.e., the mixture is less in equilibrium, as characterized by an increase in temperature at the end of the relaxation zone (Fig. 3, where a family of Hugoniot adiabats dependent on the parameter L is shown). With increase in the heat of fusion, the pressure increases, and thus the temperature of the mixture at the final equilibrium point also increases. Figures 4 and 5 show temperatures of the discrete and continuous phases along the wave (lines 1-3 for L = 2.5, 1, 0.3). With decrease in L the dependence of temperature profiles on the heat of phase transition weakens. This is because decrease in L leads to an attenuation of the phase transition process, and in the limit $L \to 0$ the mixture will have the limiting relaxation zone of a single-velocity two-temperature mixture of gas and discrete particles.

Figure 6 shows the temperature profile along the relaxation zone for several characteristic parameter values. It is evident that the gas temperature increases with braking of the mixture in the relaxation zone. The gas temperature increases more intensely due to braking. The particles heat due to heat exchange with the continuous phase, remaining colder than the gas. Increase in the relaxation time τ_1 naturally leads to freezing of the heat exchange process. At $\tau_1 \sim 0$, the change in T, T_2 to their final value occurs in a boundary layer the length of which changes little after $\tau_1 < 0.01$.

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TIME—FREQUENCY CHARACTERISTICS OF AN ELASTIC WAVE RADIATED BY A CAMOUFLET EXPLOSION

A. A. Zverev, E. E. Lovetskii, and V. S. Fetisov

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One of the features of explosive action upon a medium is the radiation of elastic waves by the explosive source. This process is of interest, since the range of action of elastic waves on the medium significantly exceeds the dimensions of the destruction zone created by the explosion. Data concerning the explosion transferred by elastic waves can be received at large distances from the center. The carrier of these data is the elastic wave, the spectrum of which is usually characterized by some fundamental frequency. The characteristic frequency is determined by the dimensions of the elastic wave source R_e: $\omega_0 = c_\ell/R_e$, where c_ℓ is the speed of sound in the perturbed medium. The spectral characteristics of the elastic wave contain information on the properties of the medium surrounding the change [1]. It is thus of interest to study the effect of medium parameters in the vicinity of the explosion on the frequency—time characteristics of the radiated wave, as well as upon the seismic efficiency of an underground explosion.

We will consider the explosive process from the moment of shock wave formation. We assume that on the shock wave front the medium is compressed due to collapse of pores. The medium then breaks into particles and behind the front the medium expands due to the dilatance effect [2]. In this stage the velocity of the shock wave front or destruction wave exceeds the speed of propagation of longitudinal compression waves in the given medium. After the velocity of the front becomes equal to the velocity of longitudinal waves elastic waves begin to radiate from the destruction wave front, continuing after the latter halts.

At the initial moment a shock wave breaks away from a spherical cavity of radius a_0, filled by gas at a pressure of p_0. The increase in density of the medium at the front is defined by the quantity [3]

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